11.1 Areas of Triangles and Parallelograms

|	
Before	You learned properties of triangles and parallelograms.
Now	You will find areas of triangles and parallelograms.
Why?	So you can plan a jewelry making project, as in Ex. 44.

Key Vocabulary
- **bases of a parallelogram**
- **height of a parallelogram**
- **area,** *p. 49*
- **perimeter,** *p. 49*

POSTULATES   *For Your Notebook*

POSTULATE 24 Area of a Square Postulate

The area of a square is the square of the length of its side.

$A = s^2$
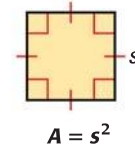
POSTULATE 25 Area Congruence Postulate

If two polygons are congruent, then they have the same area.

POSTULATE 26 Area Addition Postulate

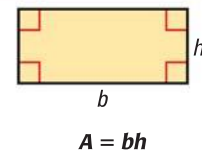The area of a region is the sum of the areas of its nonoverlapping parts.

RECTANGLES A rectangle that is *b* units by *h* units can be split into *b* · *h* unit squares, so the area formula for a rectangle follows from Postulates 24 and 26.

THEOREM   *For Your Notebook*

THEOREM 11.1 Area of a Rectangle

The area of a rectangle is the product of its base and height.

Justification: Ex. 46, p. 726

$A = bh$

READ DIAGRAMS

The word *base* can refer to a segment or to its length. The segment used for the height must be perpendicular to the bases used.

PARALLELOGRAMS Either pair of parallel sides can be used as the **bases** of a parallelogram. The **height** is the perpendicular distance between these bases.

If you transform a rectangle to form other parallelograms with the same base and height, the area stays the same.
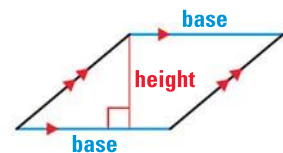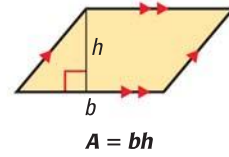
Animated **Geometry** at classzone.com

For Your Notebook

THEOREM 11.2 Area of a Parallelogram

The area of a parallelogram is the product of a base and its corresponding height.
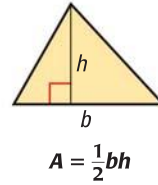
Justification: Ex. 42, p. 725

$$A = bh$$

THEOREM 11.3 Area of a Triangle

The area of a triangle is one half the product of a base and its corresponding height.

Justification: Ex. 43, p. 726

$$A = \frac{1}{2}bh$$

RELATING AREA FORMULAS As illustrated below, the area formula for a parallelogram is related to the formula for a rectangle, and the area formula for a triangle is related to the formula for a parallelogram. You will write a justification of these relationships in Exercises 42 and 43 on pages 725–726.
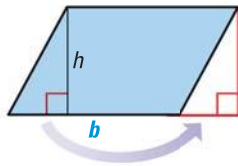
Area of ▱ = Area of Rectangle
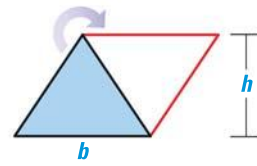
Area of △ = $\frac{1}{2}$ • Area of ▱

EXAMPLE 1   **Use a formula to find area**
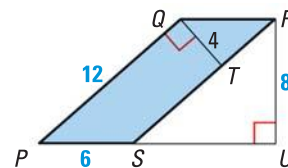
Find the area of ▱PQRS.

Solution

Method 1 Use $\overline{PS}$ as the base.
The base is extended to measure the height RU. So, $b = 6$ and $h = 8$.
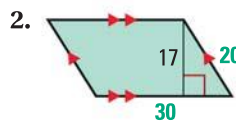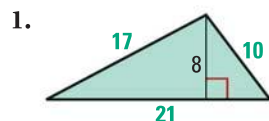
Area $= bh = 6(8) = 48$ square units

Method 2 Use $\overline{PQ}$ as the base.
Then the height is QT. So, $b = 12$ and $h = 4$.

Area $= bh = 12(4) = 48$ square units

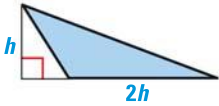✔ **GUIDED PRACTICE**   **for Example 1**

Find the perimeter and area of the polygon.

1.

2.

3.

EXAMPLE 2   Solve for unknown measures

DRAW DIAGRAMS
Note that there are other ways you can draw the triangle described in Example 2.

ALGEBRA The base of a triangle is twice its height. The area of the triangle is 36 square inches. Find the base and height.

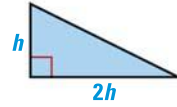Let h represent the height of the triangle. Then the base is $2h$.

$$A = \frac{1}{2}bh \qquad \text{Write formula.}$$

$$36 = \frac{1}{2}(2h)(h) \qquad \text{Substitute 36 for } A \text{ and } 2h \text{ for } b.$$

$$36 = h^2 \qquad \text{Simplify.}$$

$$6 = h \qquad \text{Find positive square root of each side.}$$

▶ The height of the triangle is 6 inches, and the base is 6 · 2 = 12 inches.

EXAMPLE 3   Solve a multi-step problem

PAINTING You need to buy paint so that you can paint the side of a barn. A gallon of paint covers 350 square feet. How many gallons should you buy?

Solution

You can use a right triangle and a rectangle to approximate the area of the side of the barn.

ANOTHER WAY
In Example 3, you have a 45°-45°-90° triangle, so you can also find x by using trigonometry or special right angles.

STEP 1 **Find** the length x of each leg of the triangle.

$$26^2 = x^2 + x^2 \qquad \text{Use Pythagorean Theorem.}$$

$$676 = 2x^2 \qquad \text{Simplify.}$$

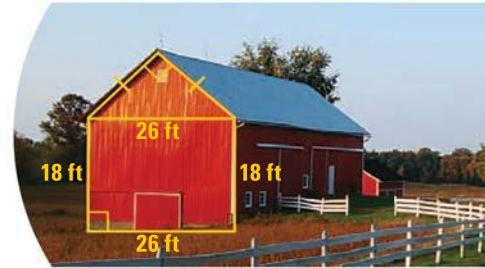$$\sqrt{338} = x \qquad \text{Solve for the positive value of } x.$$

STEP 2 **Find** the approximate area of the side of the barn.

Area = **Area of rectangle** + **Area of triangle**

$$= \mathbf{26(18)} + \frac{1}{2} \cdot \left[\left(\sqrt{338} \right) \left(\sqrt{338} \right) \right] = 637 \text{ ft}^2$$

STEP 3 **Determine** how many gallons of paint you need.

$$637 \text{ ft}^2 \cdot \frac{1 \text{ gal}}{350 \text{ ft}^2} \approx 1.82 \text{ gal} \qquad \text{Use unit analysis.}$$

▶ Round up so you will have enough paint. You need to buy 2 gallons of paint.

✓ **GUIDED PRACTICE**   for Examples 2 and 3

4. A parallelogram has an area of 153 square inches and a height of 17 inches. What is the length of the base?

5. **WHAT IF?** In Example 3, suppose there is a 5 foot by 10 foot rectangular window on the side of the barn. What is the approximate area you need to paint?

11.1 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Either pair of parallel sides of a parallelogram can be called its __?__, and the perpendicular distance between these sides is called the __?__.

2. ★ **WRITING** What are the two formulas you have learned for the area of a rectangle? *Explain* why these formulas give the same results.
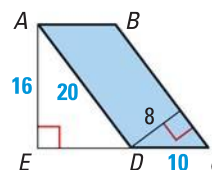
EXAMPLE 1
on p. 721
for Exs. 3–15

FINDING AREA Find the area of the polygon.

3.

4

7

4.

12

14

5.

15

6.

13 10

7.

30 18

8.

9

15
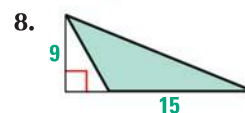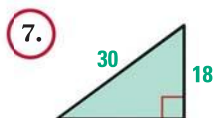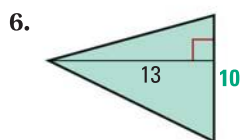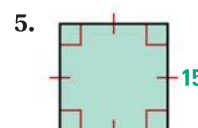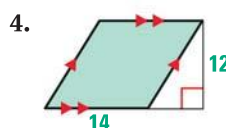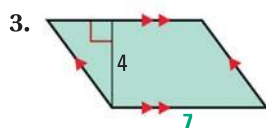
9. **COMPARING METHODS** Show two different ways to calculate the area of parallelogram *ABCD*. *Compare* your results.

A   B
16 20
8
E   D 10 C

ERROR ANALYSIS *Describe* and correct the error in finding the area of the parallelogram.

10.
$A = bh$
$= (6)(5)$
$= 30$

5   4
6

11.
$A = bh$
$= (7)(4)$
$= 28$

4
4   3

PYTHAGOREAN THEOREM The lengths of the hypotenuse and one leg of a right triangle are given. Find the perimeter and area of the triangle.
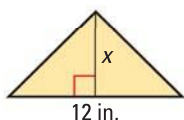
12. Hypotenuse: 15 in.; leg: 12 in.

13. Hypotenuse: 34 ft; leg: 16 ft

14. Hypotenuse: 85 m; leg: 84 m
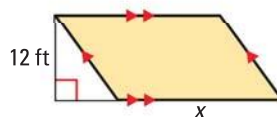
15. Hypotenuse: 29 cm; leg: 20 cm

EXAMPLE 2
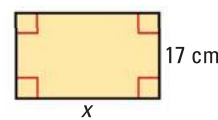on p. 722
for Exs. 16–21

xy **ALGEBRA** Find the value of *x*.

16. $A = 36$ in.^{2}

x
12 in.
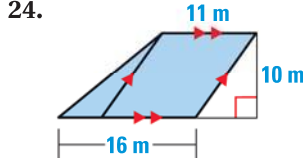
17. $A = 276$ ft^2
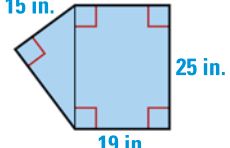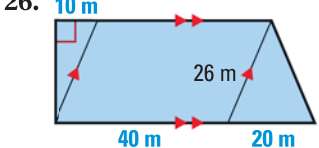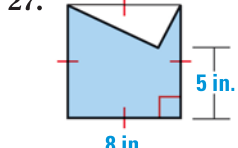
12 ft
x

18. $A = 476$ cm^2

17 cm
x
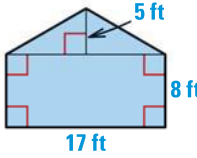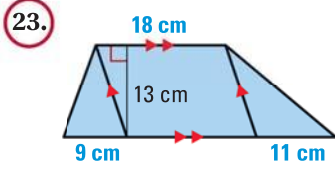
19. ⓧⓨ **ALGEBRA** The area of a triangle is 4 square feet. The height of the triangle is half its base. Find the base and the height.

20. ⓧⓨ **ALGEBRA** The area of a parallelogram is 507 square centimeters, and its height is three times its base. Find the base and the height.

21. ★ **OPEN-ENDED MATH** A polygon has an area of 80 square meters and a height of 10 meters. Make scale drawings of three different triangles and three different parallelograms that match this description. Label the base and the height.

FINDING AREA Find the area of the shaded polygon.

22.

5 ft

8 ft

17 ft

23.

18 cm

13 cm

9 cm   11 cm

24.

11 m

10 m

16 m

25.

15 in.

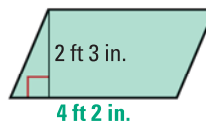25 in.

19 in.

26.

10 m

26 m

40 m   20 m

27.

5 in.

8 in.

COORDINATE GRAPHING Graph the points and connect them to form a polygon. Find the area of the polygon.

28. $A(3, 3)$, $B(10, 3)$, $C(8, -3)$, $D(1, -3)$ **29.** $E(-2, -2)$, $F(5, 1)$, $G(3, -2)$

30. ★ **MULTIPLE CHOICE** What is the area of the parallelogram shown at the right?

Ⓐ 8 ft^2 6 in.^{2} Ⓑ 1350 in.

Ⓒ 675 in.^{2} Ⓓ 9.375 ft^2

2 ft 3 in.

4 ft 2 in.

31. **TECHNOLOGY** Use geometry drawing software to draw a line ℓ and a line m parallel to ℓ. Then draw $\triangle ABC$ so that C is on line ℓ and $\overline{AB}$ is on line m. Find the base AB, the height CD, and the area of $\triangle ABC$. Move point C to change the shape of $\triangle ABC$. What do you notice about the base, height, and area of $\triangle ABC$?

32. **USING TRIGONOMETRY** In $\square ABCD$, base AD is 15 and AB is 8. What are the height and area of $\square ABCD$ if $m\angle DAB$ is 20°? if $m\angle DAB$ is 50°?

33. ⓧⓨ **ALGEBRA** Find the area of a right triangle with side lengths 12 centimeters, 35 centimeters, and 37 centimeters. Then find the length of the altitude drawn to the hypotenuse.

34. ⓧⓨ **ALGEBRA** Find the area of a triangle with side lengths 5 feet, 5 feet, and 8 feet. Then find the lengths of all three altitudes of the triangle.

35. **CHALLENGE** The vertices of quadrilateral $ABCD$ are $A(2, -2)$, $B(6, 4)$, $C(-1, 5)$, and $D(-5, 2)$. Without using the Distance Formula, find the area of $ABCD$. Show your steps.

36. SAILING Sails A and B are right triangles. The lengths of the legs of Sail A are 65 feet and 35 feet. The lengths of the legs of Sail B are 29.5 feet and 10.5 feet. Find the area of each sail to the nearest square foot. About how many times as great is the area of Sail A as the area of Sail B?

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37. MOWING You can mow 10 square yards of grass in one minute. How long does it take you to mow a triangular plot with height 25 yards and base 24 yards? How long does it take you to mow a rectangular plot with base 24 yards and height 36 yards?

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38. CARPENTRY You are making a table in the shape of a parallelogram to replace an old 24 inch by 15 inch rectangular table. You want the areas of two tables to be equal. The base of the parallelogram is 20 inches. What should the height be?

39. ★ SHORT RESPONSE A *4 inch square* is a square that has a side length of 4 inches. Does a 4 inch square have an area of 4 square inches? If not, what size square does have an area of 4 square inches? *Explain*.

40. PAINTING You are earning money by painting a shed. You plan to paint two sides of the shed today. Each of the two sides has the dimensions shown at the right. You can paint 200 square feet per hour, and you charge $20 per hour. How much will you get paid for painting those two sides of the shed?

30°
6.5 ft
12 ft
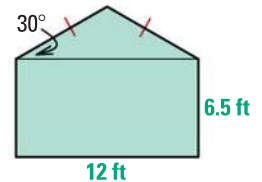
41. ENVELOPES The pattern below shows how to make an envelope to fit a card that is 17 centimeters by 14 centimeters. What are the dimensions of the rectangle you need to start with? What is the area of the paper that is actually used in the envelope? of the paper that is cut off?

14 cm
17 cm

6 cm
card fits here
14 cm
fold line
14 cm
3 cm   17 cm   3 cm

14 cm
17 cm
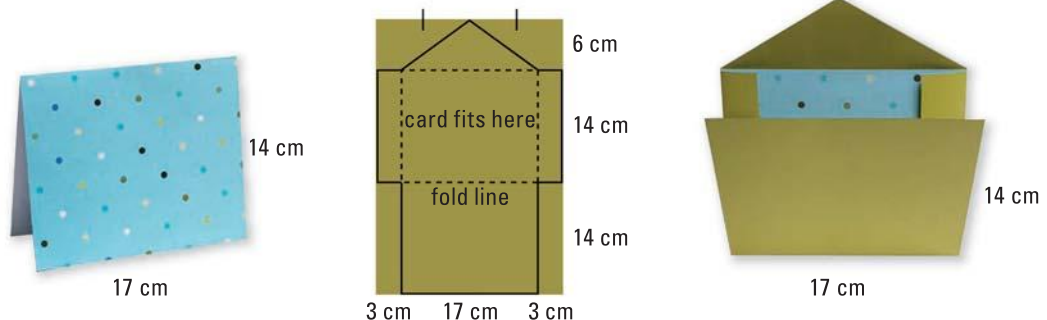
42. JUSTIFYING THEOREM 11.2 You can use the area formula for a rectangle to justify the area formula for a parallelogram. First draw ▱*PQRS* with base *b* and height *h*, as shown. Then draw a segment perpendicular to $\overleftrightarrow{PS}$ through point *R*. Label point *V*.

a. In the diagram, *explain* how you know that $\triangle PQT \cong \triangle SRV$.

b. *Explain* how you know that the area of *PQRS* is equal to the area of *QRVT*. How do you know that Area of *PQRS* = *bh*?

Q   R
h
P   T   b   S   V
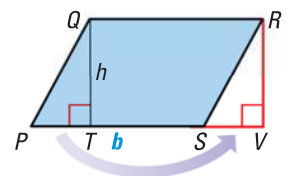
43. JUSTIFYING THEOREM 11.3 You can use the area formula for a parallelogram to justify the area formula for a triangle. Start with two congruent triangles with base *b* and height *h*. Place and label them as shown. *Explain* how you know that *XYZW* is a parallelogram and that Area of $\triangle XYW = \frac{1}{2}bh$.
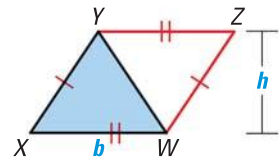
44. MULTI-STEP PROBLEM You have enough silver to make a pendant with an area of 4 square centimeters. The pendant will be an equilateral triangle. Let *s* be the side length of the triangle.

  a. Find the height *h* of the triangle in terms of *s*. Then write a formula for the area of the triangle in terms of *s*.

  b. Find the side length of the triangle. Round to the nearest centimeter.
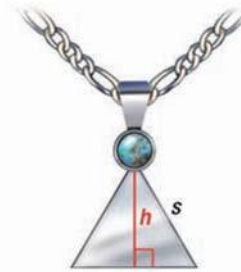
45. ★ **EXTENDED RESPONSE** The base of a parallelogram is 7 feet and the height is 3 feet. *Explain* why the perimeter cannot be determined from the given information. Is there a least possible perimeter for the parallelogram? Is there a greatest possible perimeter? *Explain*.

46. JUSTIFYING THEOREM 11.1 You can use the diagram to show that the area of a rectangle is the product of its base *b* and height *h*.
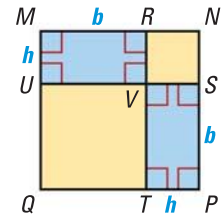
  a. Figures *MRVU* and *VSPT* are congruent rectangles with base *b* and height *h*. *Explain* why *RNSV*, *UVTQ*, and *MNPQ* are squares. Write expressions in terms of *b* and *h* for the areas of the squares.

  b. Let *A* be the area of *MRVU*. Substitute *A* and the expressions from part (a) into the equation below. Solve to find an expression for *A*.

  Area of *MNPQ* = Area of *MRVU* + Area of *UVTQ* + Area of *RNSV* + Area of *VSPT*

47. CHALLENGE An equation of $\overleftrightarrow{AB}$ is *y = x*. An equation of $\overleftrightarrow{AC}$ is *y = 2*. Suppose $\overleftrightarrow{BC}$ is placed so that $\triangle ABC$ is isosceles with an area of 4 square units. Find two different lines that fit these conditions. Give an equation for each line. Is there another line that could fit this requirement for $\overleftrightarrow{BC}$? *Explain*.
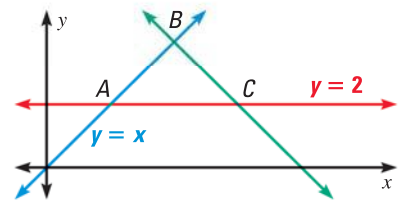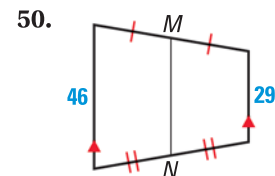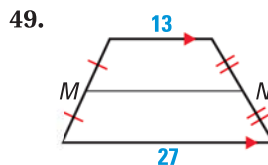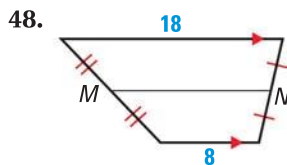

MIXED REVIEW

Find the length of the midsegment $\overline{MN}$ of the trapezoid. *(p. 542)*

48. 18 ... *M* ... *N* ... 8

49. 13 ... *M* ... *N* ... 27

50. *M* ... 46 ... 29 ... *N*

The coordinates of $\triangle PQR$ are P(−4, 1), Q(2, 5), and R(1, −4). Graph the image of the triangle after the translation. Use prime notation. *(p. 572)*

51. $(x, y) \rightarrow (x + 1, y + 4)$

52. $(x, y) \rightarrow (x + 3, y − 5)$

53. $(x, y) \rightarrow (x − 3, y − 2)$

54. $(x, y) \rightarrow (x − 2, y + 3)$